Novel formulations of CKM matrix renormalization

Bernd A. Kniehl* and Alberto Sirlin†

*II. Institut für Theoretische Physik, Universität Hamburg, Luruper Chaussee 149, 22761 Hamburg, Germany
†Department of Physics, New York University, 4 Washington Place, New York, NY 10003, USA

Abstract. We review two recently proposed on-shell schemes for the renormalization of the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix in the Standard Model. One first constructs gauge-independent mass counterterm matrices for the up- and down-type quarks complying with the hermiticity of the complete mass matrices. Diagonalization of the latter then leads to explicit expressions for the CKM counterterm matrix, which are gauge independent, preserve unitarity, and lead to renormalized amplitudes that are non-singular in the limit in which any two quarks become mass degenerate. One of the schemes also automatically satisfies flavor democracy.

Keywords: Perturbation theory, renormalization, electroweak interactions, quark flavor mixing
PACS: 11.10.Gh, 12.15.Ff, 12.15.Lk, 13.38.Be

INTRODUCTION

An important problem associated with the Cabibbo-Kobayashi-Maskawa (CKM) matrix is its renormalization. An early discussion [1] focused on the renormalization of ultraviolet (UV) divergences in the two-generation framework. Since the CKM matrix is one of the fundamental cornerstones of the weak interactions and, by extension, of the SM, it is important to develop renormalization schemes that deal with both the finite and divergent contributions with well-defined renormalization conditions. Over the last twenty years there have been several papers that address this basic problem at various levels of generality and complexity. In this talk I will describe two recently proposed on-shell renormalization schemes that we think have desirable theoretical properties [2, 3, 4]. The main difficulty arises from external-leg mixing corrections of the type depicted in Fig. 1.

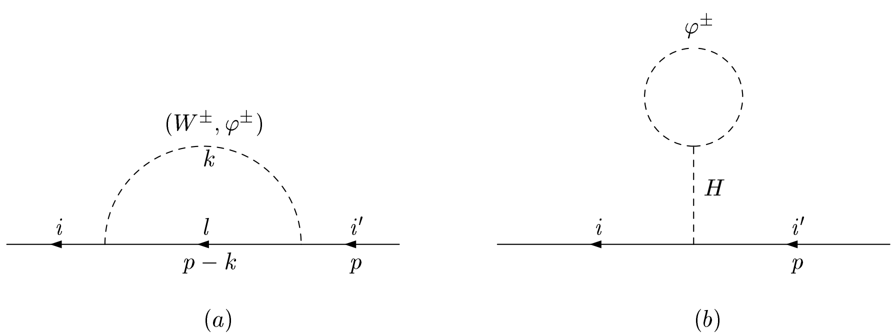

FIGURE 1. Fermion self-energy diagrams.

CKM RENORMALIZATION SCHEME OF REFS. [2, 3]

Our first proposal is a generalization of Feynman's approach in QED [5]. Recall that in QED the self-energy contribution to an outgoing fermion is given by

$$\Delta\mathscr{M}^{\text{leg}} = \overline{u}(p)\Sigma(\not{p})\frac{1}{\not{p}-m}, \tag{1}$$

$$\Sigma(\not{p}) = A + B(\not{p}-m) + \Sigma_{\text{fin}}(\not{p}), \tag{2}$$

where $\Sigma(\not{p})$ is the self-energy, A and B are divergent constants, and $\Sigma_{\text{fin}}(\not{p})$ is a finite part which is proportional to $(\not{p}-m)^2$ in the vicinity of $\not{p}=m$ and, therefore, vanishes when inserted in Eq. (1). The contribution of A to $\Delta\mathscr{M}^{\text{leg}}$ is singular at $\not{p}=m$ and is gauge independent, that of B is regular but gauge dependent. They are called self-mass (sm) and wave-function renormalization (wfr) contributions. A is cancelled by the mass counterterm δm, B is combined with proper vertex diagrams leading to a finite and gauge-independent physical amplitude. $\Sigma_{\text{fin}}(\not{p})$ does not contribute to $\Delta\mathscr{M}^{\text{leg}}$.

In the CKM case we encounter not only diagonal terms as in QED but also off-diagonal contributions generated by the diagram in Fig. 1(a). The self-energy corrections to an external quark leg are now

$$\Delta\mathscr{M}^{\text{leg}}_{ii'} = \overline{u}_i(p)\Sigma_{ii'}(\not{p})\frac{1}{\not{p}-m_{i'}}, \tag{3}$$

where i denotes the external quark of momentum p and mass m_i, and i' the virtual quark of mass $m_{i'}$.

We focus on the contributions to Eq. (3) from Fig. 1. Using a simple algorithm that treats i and i' on an equal footing, we group the contributions of Fig. 1 in four classes: (i) terms with a left factor $(\not{p}-m_i)$; (ii) terms with a right factor $(\not{p}-m_{i'})$; (iii) terms with a left factor $(\not{p}-m_i)$ and a right factor $(\not{p}-m_{i'})$; and (iv) constant terms not involving $\not{p}$. When inserted in Eq. (3), terms of class (iii) vanish. Terms of classes (i) and (ii) combine to cancel the propagator $(\not{p}-m_{i'})^{-1}$ in both diagonal and off-diagonal contributions. In analogy with B in QED, they are identified with wfr contributions. Using the unitarity relations of the CKM elements, one finds that they satisfy an important property: all the gauge-dependent and all the UV-divergent wfr contributions to the $W \to q_i + \overline{q}_j$ amplitude depend only on an overall factor V_{ij} and the external-quark masses m_i and m_j, a property shared by the one-loop proper vertex diagrams. This implies that, aside from the sm contributions to be discussed later, the proof of gauge independence and UV finiteness of the one-loop corrections to the $W \to q_i + \overline{q}_j$ amplitude is the same as in the unmixed, single-generation case!

In contrast, terms of class (iv) lead to a multiple of $(\not{p}-m_{i'})^{-1}$ with a cofactor that involves the chiral projectors $a_\pm = (1\pm\gamma_5)/2$, but not $\not{p}$. In analogy with A in QED, they are gauge independent. We identify them with the sm contributions.

Next we consider the cancellation of the sm contributions with mass counterterms

$$\overline{\psi}^Q\left(\delta m^{Q(+)}a_+ + \delta m^{Q(-)}a_-\right)\psi^Q \qquad (Q=U,D), \tag{4}$$

where $\delta m^{Q(\pm)}$ are non-diagonal matrices subject to the hermiticity condition $\delta m^{Q(+)} = \delta m^{Q(-)\dagger}$, which implies

$$\delta m^{U(+)}_{i'i} = \delta m^{U(-)*}_{ii'}, \qquad \delta m^{U(-)}_{i'i} = \delta m^{U(+)*}_{ii'} \tag{5}$$

for U quarks, and similarly for D quarks (for which we use jj'). The UV-divergent sm contributions obey the hermiticity condition, so they can be cancelled by the $\delta m^{Q(\pm)}$ in all channels. However, this is not the case for some of the finite parts. For this reason, we use a specific renormalization prescription: we adjust the $\delta m^{Q(\pm)}$ to fully cancel the sm terms in all diagonal channels, as well as the uc, ut, and ct channels for U quarks and sd, bd, and bs channels for D quarks. This implies that there are residual sm contributions in the reverse cu, tu, tc, ds, db, and sb channels, but they are finite, gauge independent and very small.

An alternative formulation is obtained by diagonalizing the complete mass matrices $m^Q - \delta m^{Q(+)} a_+ - \delta m^{Q(-)} a_-$ by unitary transformations on the ψ^Q_R and ψ^Q_L fields. Such transformations induce a CKM counterterm matrix

$$\delta V_{ij} = \sum_{i' \neq i} \frac{m^U_i \delta m^{U(-)}_{ii'} + \delta m^{U(+)}_{ii'} m^U_{i'}}{\left(m^U_i\right)^2 - \left(m^U_{i'}\right)^2} V_{i'j} - \sum_{j' \neq j} V_{ij'} \frac{m^D_{j'} \delta m^{D(-)}_{j'j} + \delta m^{D(+)}_{j'j} m^D_j}{\left(m^D_{j'}\right)^2 - \left(m^D_j\right)^2}. \tag{6}$$

In this basis, in which the complete mass matrices are diagonal, the off-diagonal sm contributions are cancelled by δV. δV automatically satisfies the following important properties: it is gauge independent, preserves unitarity in the sense that both V and $V - \delta V$ are unitary, and leads to renormalized amplitudes that are non-singular in the limit $m^U_{i'} \to m^U_i$ or $m^D_{j'} \to m^D_j$.

In summary, this approach is based on a novel procedure to separate the external-leg mixing corrections into gauge-independent sm and gauge-dependent wfr contributions, and to implement the renormalization of the former with non-diagonal mass counterterm matrices. Diagonalization of the complete mass matrices leads to an explicit CKM counterterm matrix δV, which is gauge independent, preserves unitarity, and leads to renormalized amplitudes that are non-singular in the limit in which any two quarks become mass degenerate.

It has been recently generalized to the case of an extended lepton sector that includes Dirac and Majorana neutrinos in the framework of the seesaw mechanism [6].

CKM RENORMALIZATION SCHEME OF REF. [4]

Our second approach is based on a gauge-independent quark mass counterterm that is directly expressed in terms of the Lorentz-invariant self-energy functions.

On covariance grounds, the self-energy $\Sigma_{ii'}(\not{p})$ associated with Fig. 1 is of the form

$$\Sigma_{ii'}(\not{p}) = \not{p} a_- \Sigma^L_{ii'}(p^2) + \not{p} a_+ \Sigma^R_{ii'}(p^2) + a_- A^L_{ii'}(p^2) + a_+ A^R_{ii'}(p^2). \tag{7}$$

When i is an outgoing U quark, the proposed mass counterterm is

$$\delta m_{ii'} \quad = \quad V_{il} V^\dagger_{li'} \,\mathrm{Re} \left\{ a_+ \left[\frac{m_{i'}}{2} \tilde{\Sigma}^L_{ii'}(m_i^2) + \frac{m_i}{2} \tilde{\Sigma}^R_{ii'}(m_i^2) + \tilde{A}^R_{ii'}(m_i^2) \right] \right.$$

$$+ a_- \left[\frac{m_i}{2} \tilde{\Sigma}^L_{ii'}(m^2_{i'}) + \frac{m_{i'}}{2} \tilde{\Sigma}^R_{ii'}(m^2_{i'}) + \tilde{A}^L_{ii'}(m^2_{i'}) \right] \Big\}, \tag{8}$$

where $\tilde{\Sigma}^{L,R}_{ii'}(p^2)$ and $\tilde{A}^{L,R}_{ii'}(p^2)$ are the invariant self-energies with $V_{il}V^\dagger_{li'}$ factored out. Explicit one-loop expressions for the invariant functions are given in the literature [7]. Inserting these results in Eq. (8), we get for outgoing U quarks:

$$\begin{aligned} \delta m^{(+)}_{ii'} &= \frac{g^2}{32\pi^2} V_{il} V^\dagger_{li'} \operatorname{Re} \left\{ m_i \left(1 + \frac{m_i^2}{2m_W^2} \Delta \right) - \frac{m_{i'} m_l^2}{2m_W^2} [3\Delta + I(m_i^2, m_l) \right. \\ &\quad \left. + J(m_i^2, m_l)] + m_{i'} \left(1 + \frac{m_i^2}{2m_W^2} \right) [I(m_i^2, m_l) - J(m_i^2, m_l)] \right\}, \end{aligned} \tag{9}$$

where g is the SU(2) gauge coupling, $\Delta = 1/(n-4) + [\gamma_E - \ln(4\pi)]/2 + \ln(m_W/\mu)$ the UV divergence, n the space-time dimensionality, γ_E the Euler-Mascheroni constant, μ the 't Hooft mass scale, and

$$\{I(p^2, m_l); J(p^2, m_l)\} = \int_0^1 \mathrm{d}x \, \{1; x\} \ln \frac{m_l^2 x + m_W^2 (1-x) - p^2 x (1-x) - i\varepsilon}{m_W^2}. \tag{10}$$

$\delta m^{(-)}_{ii'}$ is obtained by interchanging $m_i \leftrightarrow m_{i'}$ between the curly brackets in Eq. (9). Now there are residual sm contributions $\Delta\mathscr{M}^{\text{res}}$ in all channels, which are finite, gauge-independent, and non-singular in the limits $m_{i'} \to m_i$ or $m_{j'} \to m_j$, for $m_i, m_j < m_W$.

The mass counterterms $\delta m^{(\pm)}_{ii'}$, as well as $\delta m^{(\pm)}_{jj'}$ for D quarks, obey two important properties: (i) they are gauge independent, and (ii) they automatically satisfy the hermiticity condition of Eq. (5). This second property implies that they can be applied as they stand to all diagonal and off-diagonal amplitudes and, in this sense, they are flavor-democratic since they do not single out particular flavor channels. Again, diagonalization of the complete mass matrices leads to a gauge-independent CKM counterterm matrix δV that preserves unitarity and now satisfies another highly desirable theoretical property, namely flavor democracy.

A comparative analysis of the W-boson hadronic widths in various CKM renormalization schemes, including the ones introduced in Refs. [2, 3, 4], is presented in Ref. [8].

This work was supported in part by DFG Collaborative Research Center No. SFB 676 and NSF Grant No. PHY–0758032.

REFERENCES

1. W. J. Marciano and A. Sirlin, *Nucl. Phys.* **B 93**, 303–323 (1975).
2. B. A. Kniehl and A. Sirlin, *Phys. Rev. Lett.* **97**, 221801 1–4 (2006).
3. B. A. Kniehl and A. Sirlin, *Phys. Rev.* **D 74**, 116003 1–14 (2006).
4. B. A. Kniehl and A. Sirlin, *Phys. Lett.* **B 673**, 208–210 (2009).
5. R. P. Feynman, *Phys. Rev.* **76**, 769–789 (1949).
6. A. A. Almasy, B. A. Kniehl, and A. Sirlin, *Nucl. Phys.* **B 818**, 115–134 (2009).
7. B. A. Kniehl, *Nucl. Phys.* **B 352**, 1–26 (1991); R. Hempfling and B. A. Kniehl, *Phys. Rev.* **D 51**, 1386–1394 (1995); B. A. Kniehl, F. Madricardo, and M. Steinhauser, *Phys. Rev.* **D 62**, 073010 1–11 (2000).
8. A. A. Almasy, B. A. Kniehl, and A. Sirlin, *Phys. Rev.* **D 79**, 076007 1–5 (2009).